The Thrifty Merchant

Shylock sells rare spices by the gram. He puts iron weights of various sizes (that are certified accurate) into one or both pans of his balance, then adds spice to the upper pan until the balance becomes level. A pan can hold up to 40g of spice. (Note: iron fills less space per gram than spice does.)

Certified weights are very expensive, so Shylock owns the minimum number of these that enables him to measure any whole number of grams from 1g to 40g in a single weighing.

Shylock's balance

(a) How many certified weights does Shylock own and what does each one weigh?

(b) Shylock buys a new balance whose pans can hold up to 360g of spice. How many extra certified weights will he need to measure any whole number of grams up to 360g in a single weighing?

Answer:

(a) Shylock owns just 4 certified weights: one each of 1g, 3g, 9g, and 27g.

The table below shows how the weights 1g, 3g and 9g can be used to measure from 1g to 13g. Here L is the total of the weights put in the lower pan and U is the total of the weights put in the upper pan, so U is less than L. Then the amount of spice needed to level the balance will be L − U.

L	1	3	3	4=3+1	9	9	10=9+1	9	9	10=9+1	12=9+3	12=9+3	13=9+3+1
U		1			4=3+1	3	3	1			1		
L - U	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**

On its own, the 1g weight can measure only 1g of spice. By using the 3g weight as well, there are 3 extra possibilities: to measure 2g, subtract 1g from 3g by placing them in different pans; measuring 3g does not require the 1g weight; to measure 4g, add the 1g to the 3g by placing them in the same pan. So with a 1g and a 3g weight, there are 1+3=4 possibilities.

Now consider the further possibilities that come from using a 9g weight also. Each of the 4 possible totals that can be made from 1g and 3g can be subtracted from 9g (to give 5g up to 8g) or added to 9g (to give 10g up to 13g). In addition, 9g can be used on its own. This makes 4+4+1=9 new possibilities. So with a 1g, a 3g and a 9g weight, there are 1+3+9=13 possibilities, namely, from 1g to 13g.

Continuing in the same way, each of these 13 possibilities can be subtracted from or added to 27g, which can also be used on its own. This makes 13+13+1=27 more possibilities in which L-U takes the values from 14g to 40g.

Note that the weights are all powers of 3. This follows from the interesting fact that each power of 3 can be obtained by doubling the sum of the lower powers of 3 and adding one.

(b) To measure up to 360g of spice, Shylock will need just two extra weights in addition to those that he already owns. The weights 81g and 243g are sufficient; they would enable Shylock to measure up to 364g if his scale pans were big enough. Several other combinations will do, such as 81g and 239g.